Enhanced Student Learning in Engineering Courses with CAS Technology

Hu, Thomas W C

thomashu@ust.hk
Department of Civil Engineering
The Hong Kong University of Science and Technology

ABSTRACT

The author has taught various engineering surveying and mechanics courses that involve advanced engineering mathematics. Within a conventional teaching approach where the use of manual calculations is advocated, students enrolled in such courses may experience a heavy computational burden and associated learning difficulties. In surveying and structural analysis, for example, the standard topics of least squares and trusses both necessitate heavy algebraic work on large matrices. Although students and instructors may prefer to work with matrices that are small enough (e.g. two by two) to facilitate manual calculations, such problems may be the result of over-simplified models that cannot adequately portray physical systems. In more realistic models, ten-by-ten or even larger matrices can be expected, and their inverses may be required as an intermediate step in an examination problem. Although such mathematical tasks are only peripheral issues in engineering analysis, they often require horrendous manual labor and are practically impossible to complete by hand or with ordinary calculators. In the past, the author proposed the use of spreadsheet methods to tackle such large-scale matrix problems, with some success. However, the use of spreadsheet software is usually not possible during examinations, since computers are either not allowed or are insufficient in quantity for a large class. Thus, a more portable and efficient way of learning and tackling such problems has been developed, through the use of calculators embedded with computer algebra systems (CAS). This new way of handling matrix problems leads to a more efficient teaching and learning process, and also makes the problems more accessible to students who are less mathematically inclined or skilled.

Another main area of teaching innovation is in the analysis of beams. This is a standard topic covered in numerous elementary mechanics courses offered by civil, mechanical and aerospace engineering departments. Many instructors and textbooks still adhere to the conventional approach, in which neither the use of CAS technology nor the modeling of loads by singularity functions is advocated. In order to make the solution to beam problems more straightforward and efficient, the author started to introduce the Dirac delta function and step functions in a mechanics of materials class, where these mathematical tools are applied in conjunction with the use of CAS. This combination facilitates a most efficient analysis procedure, from modeling to the numerical/symbolic solution and visualization of results. However, the CAS approach required the use of Mathematica previously which is not available during examinations, and student interest declines when a tool is introduced yet not permitted at examination time. Therefore, although the use of CAS calculators has been implemented recently, this was not a straightforward process. As the calculators are only equipped with scaled-down versions of a fully fledged CAS used on PCs, much work had to be done to endow them with additional CAS capabilities such as integration and plotting of *singularity functions. The results are encouraging and also discussed in this presentation, along with student feedback on the teaching innovations introduced.*

Keywords

Symbolic computation, beams, least squares, matrices

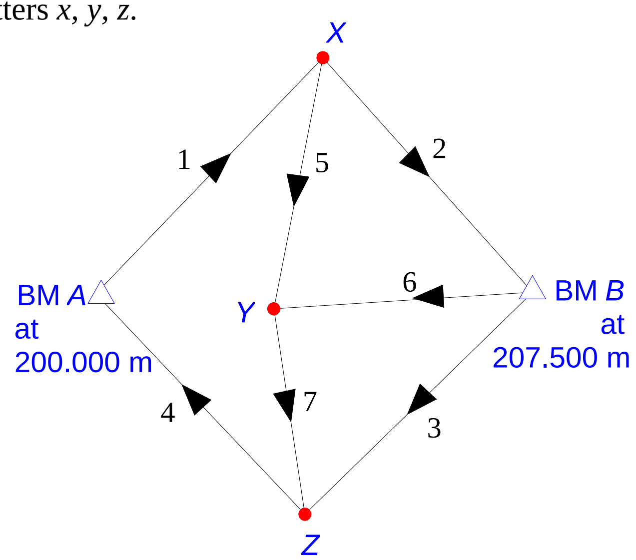
INTRODUCTION

Advanced mathematics is often used in engineering studies, and this may create a heavy burden on both teachers and students. Traditionally, manual calculations are often adopted when students handle engineering mathematics. As a result, they may experience a computational challenge and associated learning difficulties. Although proper formulation and physical interpretation should be of primary concern to engineering students while mathematical drills are peripheral issues, the latter can require much manual labor and take up disproportionately large amounts of lecture and study time. To alleviate such burdens, I have adopted computer algebra systems (CAS) in most of the classes I teach. The new CAS solution methodology implemented in my lectures leads to a more efficient teaching and learning process, and also makes engineering problems more accessible and fun for students who are not adequately trained or interested in mathematics. Teaching examples illustrating the efficient use of CAS in solving engineering problems are presented here.

USE OF CAS FOR NUMERICAL COMPUTATIONS

Least Squares Adjustment: Linear Problems

Fig. 1 shows a problem from CIVL 102 (Surveying and Surveying Camp). A leveling network is considered with arrowheads indicating the direction of leveling. For example, along line 1, there is a rise of 5.101 m from BM A to station X, i.e. $RL_X - RL_A = 5.101$, while line 3 indicates that there was a fall of 1.253 m going from B to Z, i.e. $RL_Z - RL_B = -1.253$. The unknown reduced levels (RLs) of stations X, Y, Z are denoted by respective lower-case letters *x*, *y*, *z*.



Line	Observed Elevation Difference (m)	Plan distance L (m)
1	5.101	45
2	2.342	30
3	-1.253	35
4	-6.134	30
5	-0.685	25
6	-3.006	20
7	1.707	20

Fig. 1 A Leveling Network

Weights are assigned as being inversely proportional to the plan distances L, which lead to a seven-by-seven weight matrix W with diagonal entries

$$w_i = \frac{1}{L_i}$$

for $i = 1, 2, \ldots, 7$.

The best estimates for the unknowns $\boldsymbol{x} = (x, y, z)^T$ are obtained by the least squares formula (Hu and Kuang, 2006)

$$\boldsymbol{x} = (A^T W A)^{-1} A^T W k$$

where

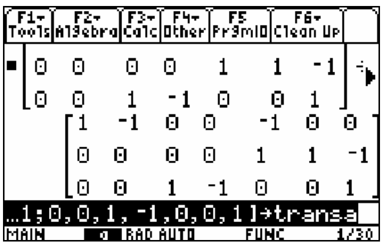
$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad k = \begin{bmatrix} 5.101 \\ 2.342 \\ -1.253 \\ -6.134 \\ -0.685 \\ -3.006 \\ 1.707 \end{bmatrix} - \begin{bmatrix} -200 \\ 207.5 \\ -207.5 \\ 200 \\ 0 \\ -207.5 \\ 0 \end{bmatrix}$$

As large matrices are involved in such problems, a manual solution can be very tedious for the student. Alternatively, this problem can be solved on the TI-89t CAS calculator (or similar calculators with matrix capabilities). First, we enter A^T (transpose of A, to be stored in variable "transa"). Noting the row-by-row entry format for matrices, we find that it is easier to enter A^T than A itself. Therefore, the transpose of A is entered first, as follows:

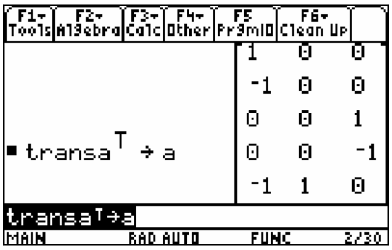
[1,(-)1,0,0,(-)1,0,0;0,0,0,0,1,1,(-)1;0,0,1,(-)1,0,0,1]STO->transa

Note:

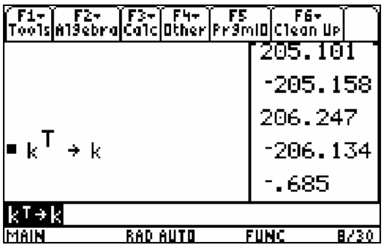
- Press [2ND] to access the blue characters such as "["above the keys
- Negation sign is entered by the [(-)] key, not the [–] key (which is for subtraction)
- Matrix elements are enclosed in a pair of square brackets, entered row by row (rows are separated by semi-colons).



Now type "transa" and use the shortcut [MATH] – [4] – [1] (for transpose) followed by [ENTER] to get the transpose of (i.e. $(A^T)^T = A$), and store the result in a, as shown next:



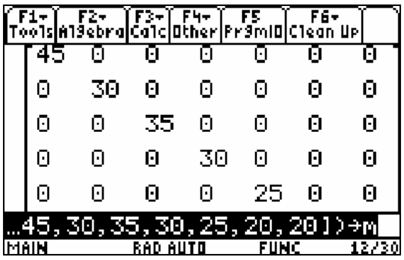
We then compute *k* as the difference of two (row) vectors, and then transpose the result into a column vector as follows:



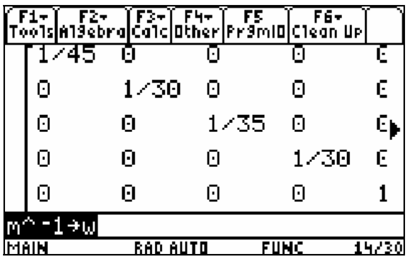
Now we enter the weight matrix efficiently as follows:

- Use shortcut MATH – 4 – 8 to enter a diagonal matrix *m* whose diagonal terms are the respective plan distances in Fig. 1:

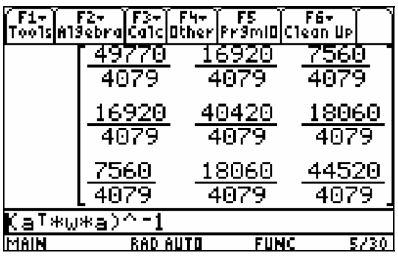
Diag([45,30,35,30,25,20,20]) STO-> m ENTER:



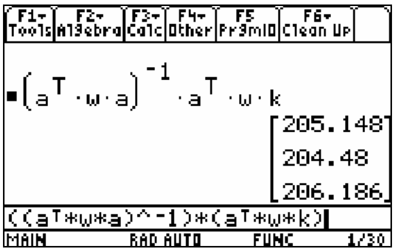
- Enter m^(-)1 to get the inverse of *m*, and store it in *w*, the weight matrix.



Calculation of $(A^TWA)^{-1}$ is efficiently performed as shown below:



Finally, we obtain the answers for *x, y, z*:



Compared to any manual approach, this calculator-based method can save a great deal of time and reduce potential errors. It also permits the discussion of more realistic problems in surveying, where the number of stations is typically large (tens or even hundreds), and it is impossible to limit the "A" and "W" matrices to only three rows or columns. However, without a CAS calculator at hand, it could be a daunting task for students to handle matrices larger than three-by-three during an examination.

Beam Analysis

CAS calculators also serve as a great tool for visualization of engineering problems. For example, in CIVL 112 (Mechanics of Materials), one often needs to plot shear and bending moment diagrams. To expedite the process, I introduced the Dirac delta function and the unit step function to express loading, shear force and bending moment functions in beam analysis. This unorthodox approach is eminently suitable when CAS is used to obtain solutions, since concentrated and distributed loads can be unified into a single load function to accelerate the solution process together with CAS. However, some additional work is needed to make the calculators recognize such functions. For example, in order to plot the shear and bending moment

$$M(x) = [-2x^2 – 48H(x – 12) + (x – 4)(2x + 13)H(x–4)]/4$$

where H(.) denotes the unit step function, we first have to program the calculator to make it understand what a step function ("h") is, as follows:

- HOME → F4 → *1:Define* → ENTER

  Define **h(x)=when(x<0,0,when(x=0,undef,1))**ENTER

- Diamond key → F1 (for "**Y =**" function input) and assign M(x) to y1 for plotting, as seen in Fig. 2(a):

  y1=**(-2x^2–48h(x–12)+(x–4)(2x+13)h(x–4))/4** ENTER

- Diamond key → F2 (for "**WINDOW**") and set up the plot area as follows:
  - xmin=**0**; xmax=**20** (beam length); xscl=**1** (value between consecutive tick marks on x-axis); ymin= **-11**; ymax=**3**; yscl=**1**; xres=**2** (resolution = 1 to 10 for decreasing fineness and increasing plot speed)

- Diamond key → F3 (for "**GRAPH**") to graph the function, as seen in Fig. 2(b):

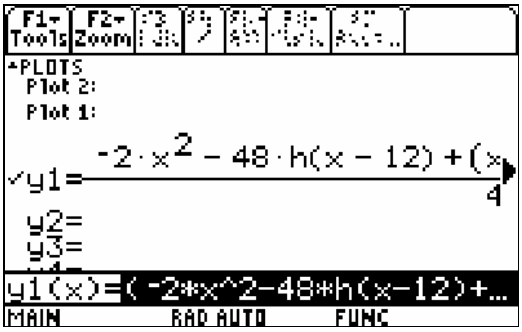


Fig. 2(a)

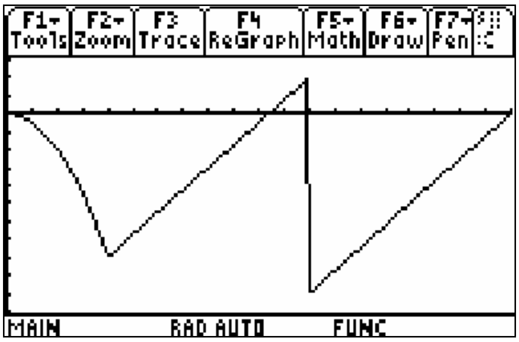


Fig. 2(b)

Learning this approach can save a great deal of time for students in engineering mechanics who have to compute and plot bending moments quite frequently.

USE OF CAS FOR SYMBOLIC COMPUTATIONS

Deflection of Beams by Energy Method

Beam deflection is one of the main topics involved in many mechanics of materials courses. There is an elegant method called Castigliano's Theorem for finding beam deflection at a point, which involves the integration of the strain energy throughout the beam, followed by differentiation of the integral with respect to the load (real or fictitious) applied at that point. As an example, consider the hinged beam (Gere 2004) in Fig. 3, where the deflection at the tip E is required:

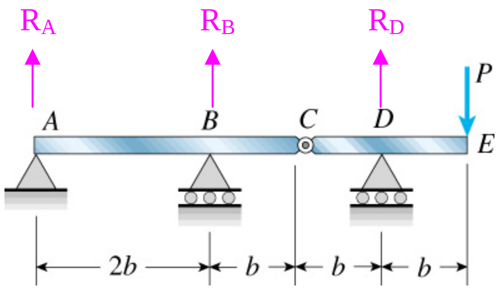


Fig. 3

Depending on the nature of the loads and hence the bending moment, the expression for the elastic energy in a beam can be quite complicated. Thus, the integration and subsequent differentiation involved can be rather long-winded and error-prone. Nevertheless, the basic principle of the energy method is mentally straightforward. In order to make the solution process as straightforward as the principle behind it, my students were taught to solve the entire problem using Mathematica (a PC-based CAS system). The entire solution process (determination of all support reactions, shear force, and bending moment; integrating the elastic energy and differentiation of the result) took only ten lines of commands, as shown in Fig. 4. It is noted that both the Dirac delta and the unit step function are available as built-in functions in Mathematica.

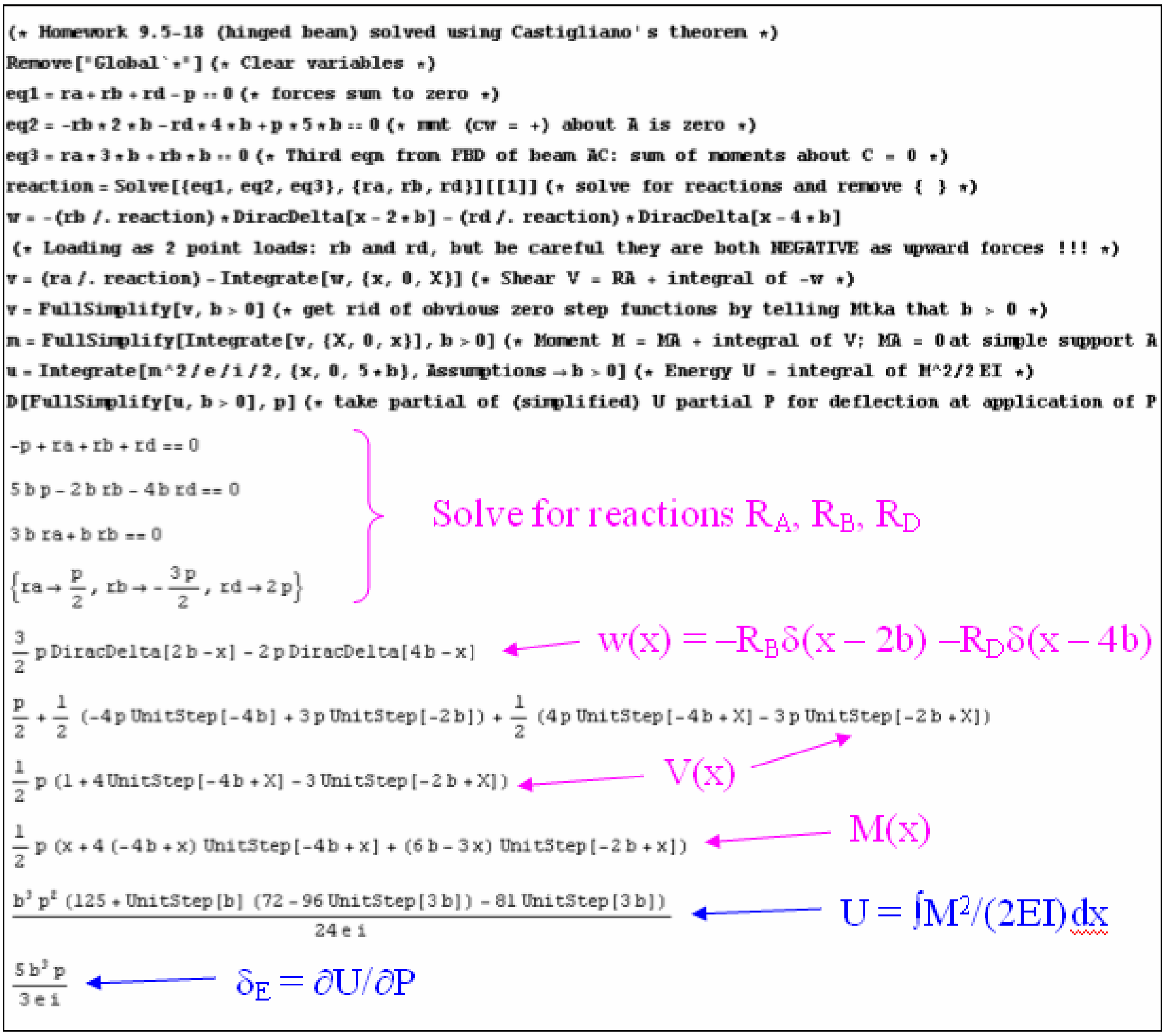


Fig. 4 Using CAS to Solve for Beam Deflection

Despite the simplicity and ease of use of CAS on a computer, such tools are usually not available during examinations. Hence, CAS calculators are also used to solve the problem to ensure that students can still benefit from this innovative approach when tackling their final exams. First, the discontinuous bending moment function is split into three parts as follows:

x =	0 to 2b	2b to 4b	4b to 5b
M(x) =	m1 = px/2	m2 = p(3b – x)	m3 = p(x – 5b)

Then the integration and differentiation are carried out in a straightforward manner, as shown in Fig. 5.

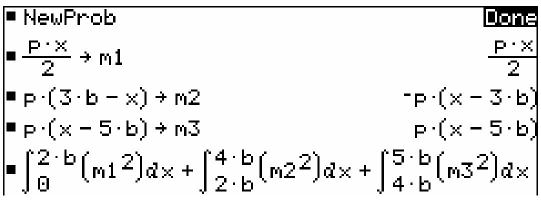


Fig. 5 Implementing Castigliano's Theorem on a CAS Calculator

The result (5Pb3/3EI) in Fig. 5 is identical to the Mathematica result in Fig. 4. The integral was split into three parts, due to the lack of integration capabilities for Dirac delta and unit step functions on CAS calculators. However, in a subsequent project, my students successfully programmed the TI-89t CAS calculator to also carry out integration of the Dirac delta and unit step functions. The team involved in this project displayed much enthusiasm when developing new functions for their CAS calculators, and they reported that their subject knowledge was further reinforced by having to program the tasks involved in the theory.

CONCLUSION

Using CAS technology in teaching and learning is akin to the use of machinery in farming to replace physical labor. Productivity is significantly enhanced, while mental labor on purely mathematical issues is kept to a minimum by delegating the mathematics chores to a CAS. Problems that are more realistic and challenging can be assigned, since the horrendous mathematics involved no longer creates a learning barrier. The time saved by using CAS can then be used to treat the engineering subject matter in more detail, focusing on the physics rather than the mathematics. The new CAS-assisted approach introduced here for teaching and learning leads to more depth and breadth, and is strongly recommended for other science and engineering courses where heavy mathematics can be expected.

ACKNOWLEDGEMENTS

The work described in this paper is supported by the CLI project, and I wish to thank the Center for Enhanced Learning and Teaching, HKUST for its generous support for this project. I also thank my students Kong Tao, Lau Yun Man and Ooi Ghee Leng for their excellent programming work for the TI-89t CAS calculator.

REFERENCES

J. M. Gere. (2004). *Mechanics of Materials*. 6th Edition. Thomson Learning.

W. C. Hu and J. S. Kuang. (2006). *Surveying for Civil Engineers: Theory and Practice with TI-89t, Excel and AutoCAD Applications.* Singapore: McGraw-Hill.