Enhanced Student Learning in Engineering Courses with CAS Technology

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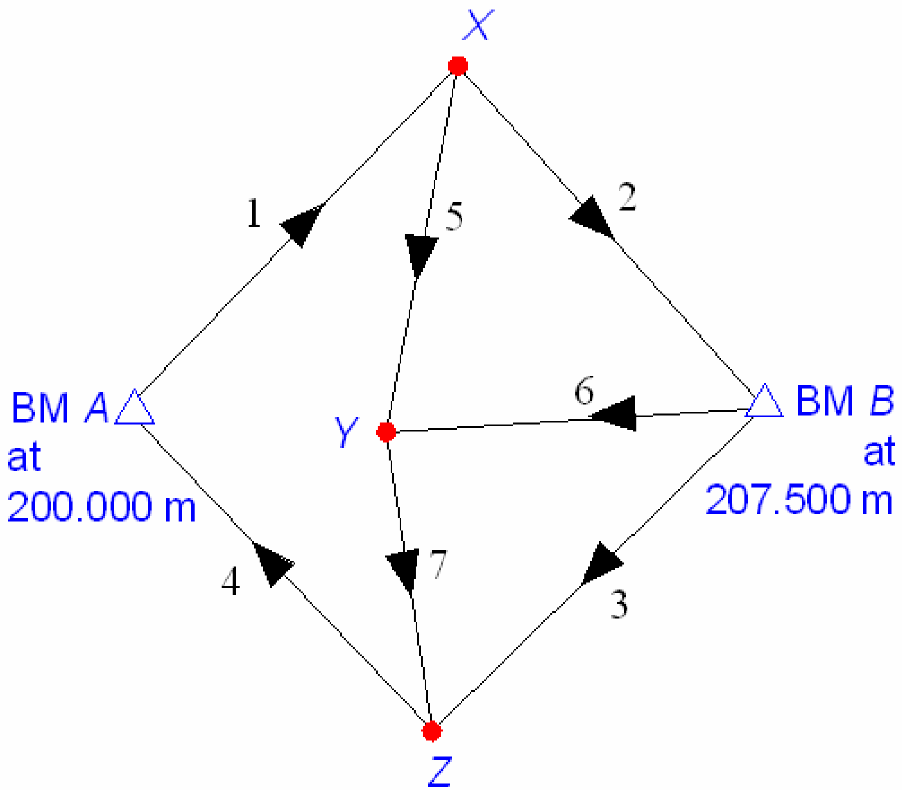
- Engineering subjects: advanced mathematics
- Burden on both teachers and students
- Traditionally: manual calculations
  - computational challenge
  - learning difficulties
- Primary concern should be proper formulation and physical interpretation
- Mathematical drills: peripheral issues, yet
  - manual labor → disproportionately large amounts of lecture/ study time

Computer Algebra Systems (CAS)

- More efficient teaching and learning process
- Problems more accessible and fun
- Examples (efficient use of CAS in solving engineering problems): surveying, mechanics of materials, statics

Numerical Computation: LSA

Least Squares Adjustment (Linear)



Line	Observed Elevation Difference (m)	Plan distance L (m)
1	5.101	45
2	2.342	30
3	-1.253	35
4	-6.134	30
5	-0.685	25
6	-3.006	20
7	1.707	20

Fig. 1 A Leveling Network

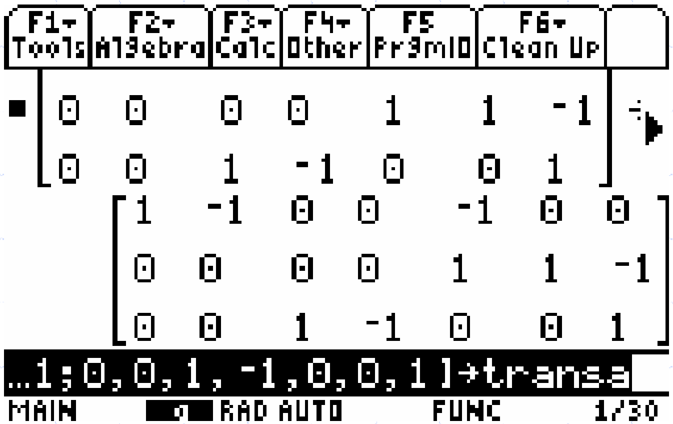
Numerical Computation: LSA

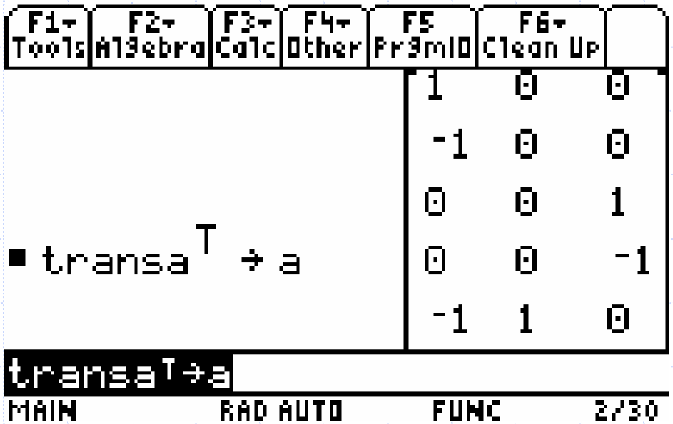
$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad k = \begin{bmatrix} 5.101 \\ 2.342 \\ -1.253 \\ -6.134 \\ -0.685 \\ -3.006 \\ 1.707 \end{bmatrix} - \begin{bmatrix} -200 \\ 207.5 \\ -207.5 \\ 200 \\ 0 \\ -207.5 \\ 0 \end{bmatrix}$$

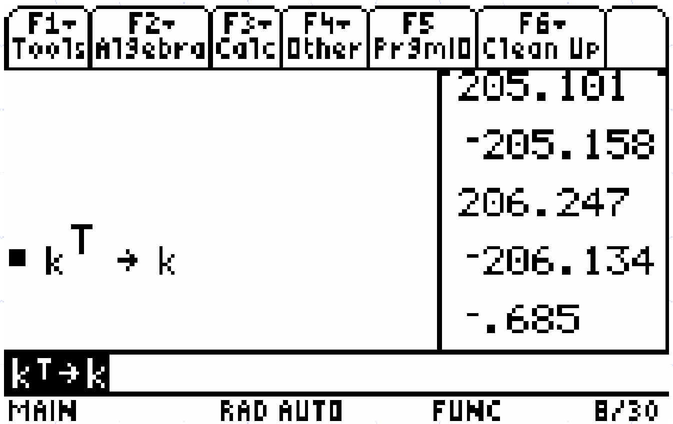
W = diagonal weight matrix

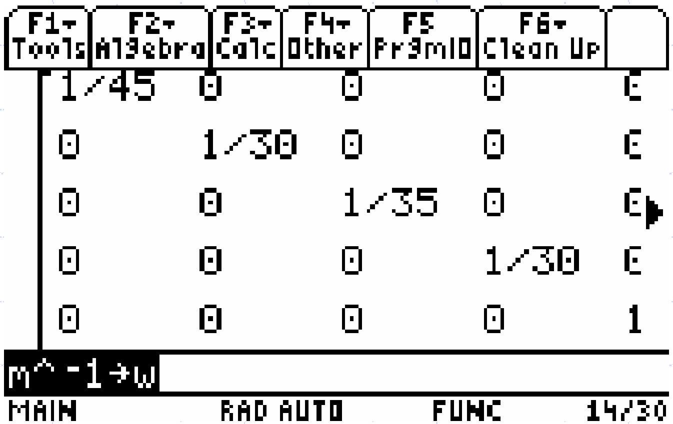
$$\boldsymbol{x} = (A^T W A)^{-1} A^T W k$$

Numerical Computation: LSA

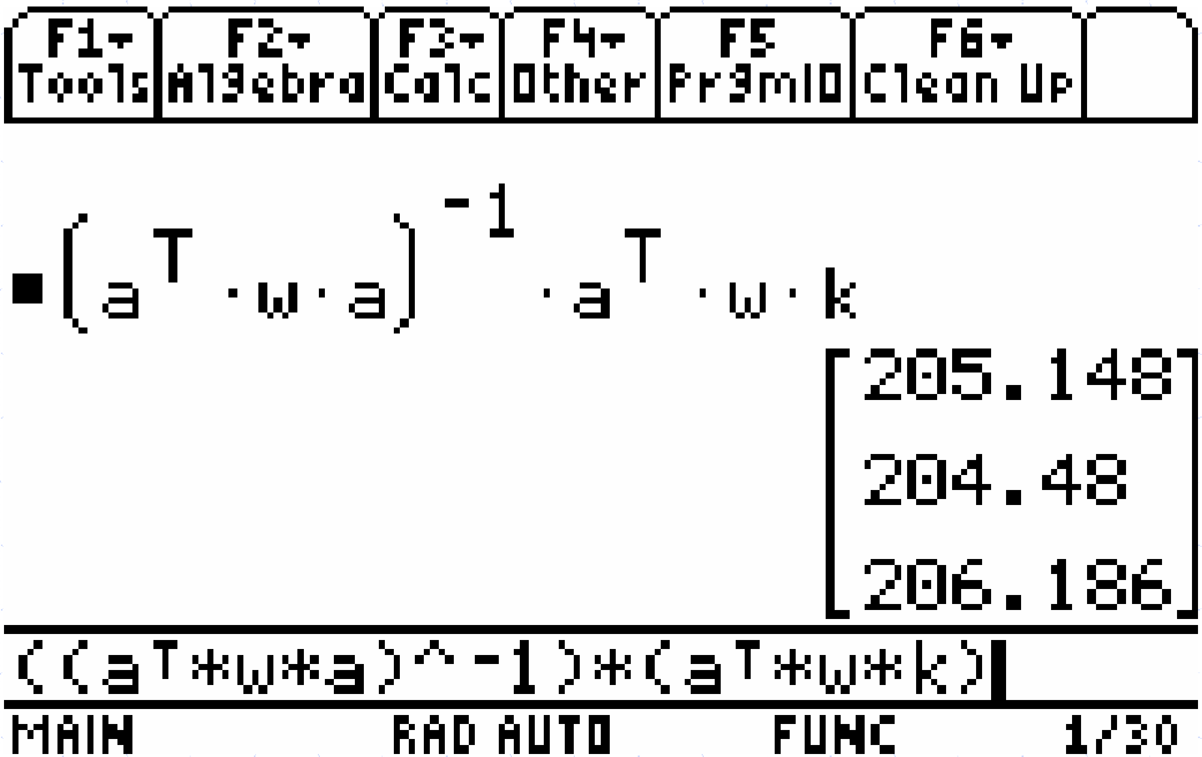






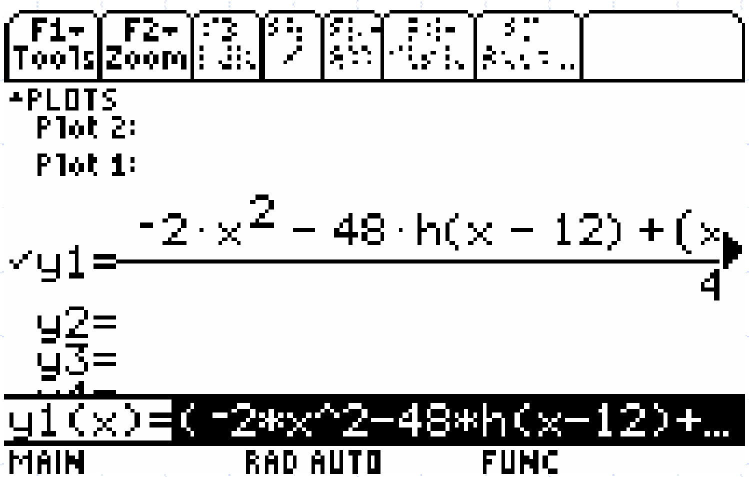


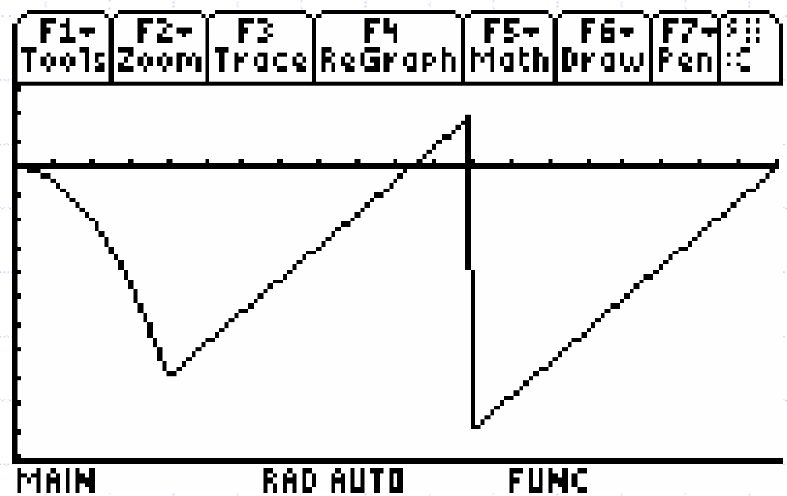
Numerical Computation: LSA



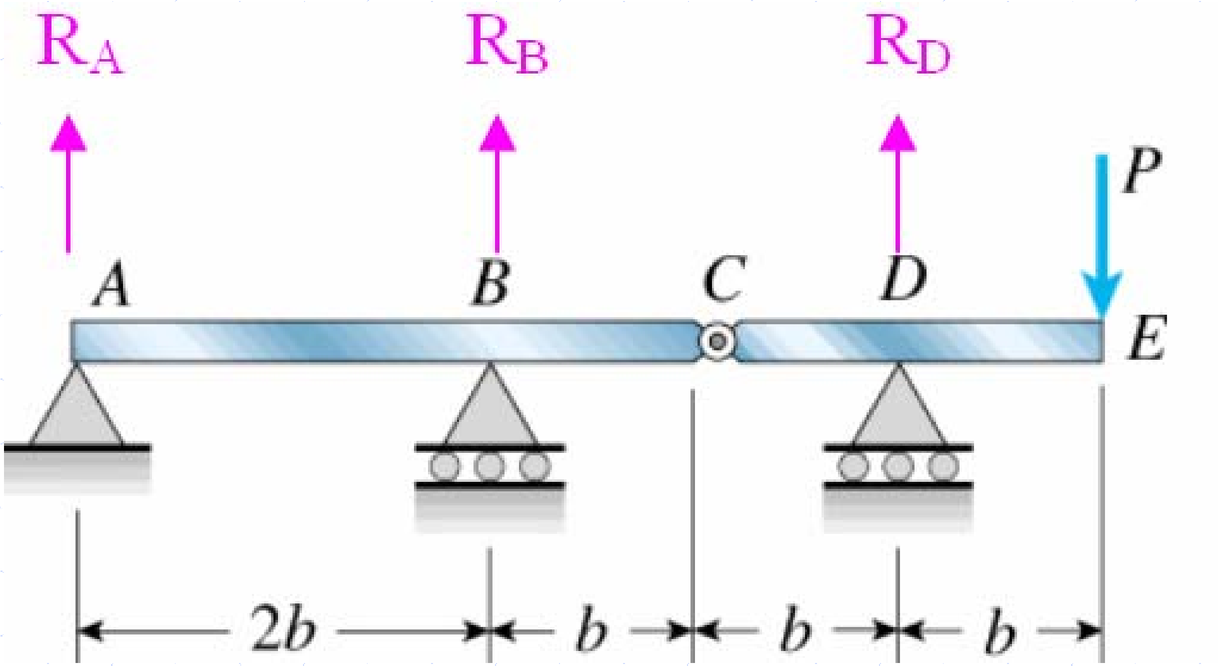
Numerical Computation: Beams

- Bending moment (singularity functions used):
$M(x) = [-2x^2 - 48H(x - 12) + (x - 4)(2x + 13)H(x - 4)]/4$
- Define **h(x)=when(x<0,0,when(x=0,undef,1))**





Symbolic Computation: Castigliano's Theorem



→ Solve for reactions, then V(x) & M(x)

→ Integrate $M(x)^2/2EI$ over the whole beam, differentiate w.r.t. load at point of interest for deflection

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(* Homework 9.5-18 (hinged beam) solved using Castigliano's theorem *)
Remove["Global`*"] (* Clear variables *)
eq1 = ra + rb + rd - p == 0 (* forces sum to zero *)
eq2 = -rb * 2 * b - rd * 4 * b + p * 5 * b == 0 (* mmt (cw = +) about A is zero *)
eq3 = ra * 3 * b + rb * b == 0 (* Third eqn from FBD of beam AC: sum of moments about C = 0 *)
reaction = Solve[{eq1, eq2, eq3}, {ra, rb, rd}][[1]] (* solve for reactions and remove { } *)
w = -(rb /. reaction) * DiracDelta[x - 2 * b] - (rd /. reaction) * DiracDelta[x - 4 * b]
 (* Loading as 2 point loads: rb and rd, but be careful they are both NEGATIVE as upward forces !!! *)
v = (ra /. reaction) - Integrate[w, {x, 0, X}] (* Shear V = RA + integral of -w *)
v = FullSimplify[v, b > 0] (* get rid of obvious zero step functions by telling Mtka that b > 0 *)
m = FullSimplify[Integrate[v, {X, 0, x}], b > 0] (* Moment M = MA + integral of V: MA = 0 at simple support A
u = Integrate[m^2 / e / i / 2, {x, 0, 5 * b}, Assumptions -> b > 0] (* Energy U = integral of M^2/2 EI *)
D[FullSimplify[u, b > 0], p] (* take partial of (simplified) U partial P for deflection at application of P
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$-p + ra + rb + rd == 0$

$5\,b\,p - 2\,b\,rb - 4\,b\,rd == 0$

$3\,b\,ra + b\,rb == 0$

$\left\{ra \to \frac{p}{2},\ rb \to -\frac{3\,p}{2},\ rd \to 2\,p\right\}$

Solve for reactions R_A, R_B, R_D

$\frac{3}{2}\,p\,\mathrm{DiracDelta}[2\,b - x] - 2\,p\,\mathrm{DiracDelta}[4\,b - x]$ ← $w(x) = -R_B\delta(x - 2b) - R_D\delta(x - 4b)$

$\frac{p}{2} + \frac{1}{2}\,(-4\,p\,\mathrm{UnitStep}[-4\,b] + 3\,p\,\mathrm{UnitStep}[-2\,b]) + \frac{1}{2}\,(4\,p\,\mathrm{UnitStep}[-4\,b + X] - 3\,p\,\mathrm{UnitStep}[-2\,b + X])$

$\frac{1}{2}\,p\,(1 + 4\,\mathrm{UnitStep}[-4\,b + X] - 3\,\mathrm{UnitStep}[-2\,b + X])$ ← $V(x)$

$\frac{1}{2}\,p\,(x + 4\,(-4\,b + x)\,\mathrm{UnitStep}[-4\,b + x] + (6\,b - 3\,x)\,\mathrm{UnitStep}[-2\,b + x])$ ← $M(x)$

$\frac{b^3\,p^2\,(125 + \mathrm{UnitStep}[b]\,(72 - 96\,\mathrm{UnitStep}[3\,b]) - 81\,\mathrm{UnitStep}[3\,b])}{24\,e\,i}$ ← $U = \int M^2/(2EI)\,dx$

$\frac{5\,b^3\,p}{3\,e\,i}$ ← $\delta_E = \partial U/\partial P$

Symbolic Computation: Castigliano's Theorem with CAS calculator

- NewProb — Done
- $\frac{p \cdot x}{2} \to m1$ — $\frac{p \cdot x}{2}$
- $p \cdot (3 \cdot b - x) \to m2$ — $^{-}p \cdot (x - 3 \cdot b)$
- $p \cdot (x - 5 \cdot b) \to m3$ — $p \cdot (x - 5 \cdot b)$
- $\int_{0}^{2 \cdot b} (m1^2)\, dx + \int_{2 \cdot b}^{4 \cdot b} (m2^2)\, dx + \int_{4 \cdot b}^{5 \cdot b} (m3^2)\, dx$ — $\frac{5 \cdot b^3 \cdot p^2}{3}$
- $\frac{\frac{\frac{5 \cdot b^3 \cdot p^2}{3}}{2}}{ei} \to u$ — $\frac{5 \cdot b^3 \cdot p^2}{6 \cdot ei}$
- $\frac{d}{dp}(u)$ — $\frac{5 \cdot p \cdot b^3}{3 \cdot ei}$

d(u,p)

MAIN DEG AUTO FUNC 7/30

- CAS calculators: cannot integrate Dirac delta or step functions
- Student project: programmed TI-89t CAS calculator to carry out such integration
- Subject knowledge was further reinforced by having to program the tasks involved in the theory

Conclusions

- CAS technology in teaching and learning ~ machinery replacing physical labor in industry/ agriculture/ etc.
- Significantly enhanced productivity
- Min. mental labor on purely math. issues
- Teach/ learn more realistic and challenging problems
- Focus on the physics rather than math
- CAS-assisted approach for other science & engineering courses with heavy mathematics